AMSAA

TECHNICAL MEMORANDUM NO. 108

SEQOPT
A SOLUTION TO THE FORCE STRUCTURING PROBLEM

Wayne S. Copes

June 1971

U.S. ARMY ABERDEEN RESEARCH AND DEVELOPMENT CENTER
ARMY MATERIEL SYSTEMS ANALYSIS AGENCY
ABERDEEN PROVING GROUND, MARYLAND

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ABSTRACT

SEQOPT has been designed to solve a specific force structuring and allocation problem. Given the following:

1. A set of targets each with a relative value and an associated single shot kill probability;

2. A set of launch points for weapons and the number of weapons allowed at each launch point;

3. The payload types available for use in these weapons;

4. The set of accessible targets for a weapon carrying each payload type from every launch point.

SEQOPT will then determine the payload type for each weapon in the force, and the allocation of each weapon to its set of accessible targets, in a way which attempts to maximize the expected damage to the target complex.

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SEQOPT
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1. INTRODUCTION

SEQOPT is a computer model written in FORTRAN IV for the BRLESC II (Ballistic Research Laboratory Electronic Scientific Computer), which chooses the payload type to be used by each weapon in a force, and allocates these weapons to a target complex in a manner whose aim is to maximize the expected damage of a target complex.

The method used in the model is called sequential optimization and does not necessarily yield a globally optimal solution. Those methods which would yield an optimal solution are at this point in time unfeasible for computer use.

2. DEFINITIONS

Before the problem can be adequately described, some terms, which will have precise meanings in this report, must be defined:

<u>Relative Target Value</u>: Each target has associated with it a relative value, which can be based upon any consistent attribute, such as industrial floor space, population, etc. (to be determined by the analyst).

<u>Weapon</u>: The weapon can be assumed to be a bomber or a variable payload missile which is capable of depositing the warheads it carries on one or more of the targets which are accessible to it.

<u>Payload Type</u>: The payload is composed of a specific number of warheads, all of which are exactly alike in characteristics. Thus payload types differ not in the types of warheads involved but in the number of warheads involved.

<u>Warhead</u>: The warhead could be either a bomb or an atomic warhead, specified by yield; the standard warhead used in all payload types.

<u>Single Shot Kill Probability (PK)</u>: The PK associated with each target is actually a function of target and attacking warhead characteristics. However, since the warhead type is fixed throughout the problem, a PK for each target can be calculated.

Launch Point: The launch point can be considered to be a bomber base or the point from which missiles are launched from a submarine. Capacity refers to the number of weapons which are to be launched from a launch point.

Accessible Targets: Associated with each weapon carrying a specific payload type is a maximum range. The accessible targets for a weapon with a specific payload from a given launch point are all those targets in the target complex lying within the maximum and minimum ranges associated with that payload type.

Outload: The outload is the choice of payload type for each weapon at each launch point, the final structure of the force.

3. MISSILE ALLOCATION PROBLEM

3.1 Description of Problem.

A great deal of work has been done in solving the so called "Missile-Allocation Problem." The problem considered is this: given an existing weapon force and set of targets, what is the optimal allocation of weapons to targets?[1] (i.e., How can we maximize expected damage to the target complex?)

The SEQOPT model is concerned with a variation of this standard allocation problem. The new problem may be described as follows: given a set of fixed launch points, the number of weapons at each launch point, and a target complex, choose the payload type for each weapon, and allocate the weapons to targets so as to maximize the expected damage to the target complex.

The obvious difference between these problems is that, in the standard allocation problem, we are given a fixed force and asked to allocate it optimally. In the force structuring problem, which SEQOPT solves, we are asked to actually structure the force, so that when it is allocated the expected damage is optimal.

[1] Matlin, Samuel, "A Review of the Literature on the Missile Allocation Problem," Operations Research 18, pp. 334-373 (1970).

An example will serve to clarify the distinction between problems, and what is meant by "structuring the force." Suppose the offense consists of one launch point which has a capacity for two weapons (bombers or missiles):

1. The standard problem would state that these weapons carry payloads consisting of N_1, and N_2 warheads, respectively. Given the targets which are accessible to weapons carrying these payloads, allocate these weapons to the target complex optimally.

2. The new problem asks: If the payloads for the two weapons can vary between A and B warheads per weapon, determine the payloads for the two weapons which, when allocated optimally, will yield the greatest expected damage to the target complex. Notice here that N_1 and N_2, as well as their allocations, are determined. $(A \leq N_1, N_2 \leq B)$.

Obviously, the expected value damaged achieved in the new problem is at least as great as that in the standard problem. This is because, in the new problem, all possible combinations of payloads are tested for return; then the outload which yields the greatest damage is chosen.

In general, it will be found that the maximum return is not achieved by carrying the "heaviest" payloads possible. This is due to the fact that, as the payload size increases, the maximum effective range of the weapon decreases under the condition of a constant quantity of propellant. Therefore, when carrying the maximum payload, some of the most valuable targets may lie out of range; consequently, some value is not damaged.

The exception to this rule exists when all targets are within range of weapons carrying the heaviest payloads from every launch point considered. In this case it is advantageous to use only the heaviest payloads available, since the use of lighter payloads makes no new targets available. However, this special case is of little interest and so we will not consider it.

Precise mathematical descriptions of the standard allocation and force structuring problems follow.

3.2 Mathematical Description of Allocation Problems.

Some definitions are needed before we can formulate the problem:

LAUNCH POINT VARIABLES

- W_i = the number of weapons allowed at the i^{th} launch point (capacity).
- LP = the number of launch points being considered.
- δ_{ij} = the number of weapons of the j^{th} payload type placed at the i^{th} launch point.
- A_{ijk} = 1 if the k^{th} target is in range of the j^{th} payload type from the i^{th} launch point; 0 otherwise.
- $\hat{A}_{ij}$ = the vector whose components are A_{ijk} for $k=1,\ldots,T$, $\hat{A}_{ij} = \left\{A_{ij1}, A_{ij2},\ldots,A_{ijT}\right\}$.

WEAPON VARIABLES

- M = the number of payload types being considered.
- α_j = the number of warheads in the j^{th} payload type.

TARGET VARIABLES

- X_{ijk} = the number of warheads from weapons of the j^{th} payload type from the i^{th} launch point, placed on the k^{th} target.
- N_k = the number of warheads placed on the k^{th} target

$$N_k = \sum_{i=1}^{LP} \sum_{j=1}^{M} X_{ijk}$$

- T = the number of targets in target complex.
- f_k = the expected damage function associated with the k^{th} target.
- PK_k = the single shot kill probability of the k^{th} target (based on target and weapon characteristics).
- V_k = the relative value of the k^{th} target.

Both allocation problems have the same formulation, the same objective function, but a slight variation in the constraints.

$$\text{Maximize } G = \sum_{k=1}^{T} f_k(V_k, PK_k, N_k)$$

Subject to:

$$\text{1st Launch Point}\left\{\begin{array}{l} X_{111} + X_{112} + \dots + X_{11T} = \delta_{11} \cdot \alpha_1 \\ X_{121} + X_{122} + \dots + X_{12T} = \delta_{12} \cdot \alpha_2 \\ \vdots \\ X_{1M1} + X_{1M2} + \dots + X_{1MT} = \delta_{1m} \cdot \alpha_M \end{array}\right\} \begin{array}{l} \text{where } \delta_{1j} = 0,1,\dots,W_1 \\ \text{and } \sum_{j=1}^{M} \delta_{1j} = W_1 \end{array}$$

$$\vdots$$

$$LP^{th}\ \text{Launch Point}\left\{\begin{array}{l} X_{LP11} + X_{LP12} + \dots + X_{LP1T} = \delta_{LP1} \cdot \alpha_1 \\ X_{LP21} + X_{LP22} + \dots + X_{LP2T} = \delta_{LP2} \cdot \alpha_2 \\ \vdots \\ X_{LPM1} + X_{LPM2} + \dots + X_{LPMT} = \delta_{LPM} \cdot \alpha_M \end{array}\right\} \begin{array}{l} \text{where } \delta_{LPj} = 0,1,2,\dots,W_{LP} \\ \text{and } \sum_{j=1}^{M} \delta_{LPj} = W_{LP} \end{array}$$

All $X_{ijk} \geq 0$, and if $A_{ijk} = 0$, then $X_{ijk} = 0$.

The entire difference between the two problems lies in the δ_{ij} variables, which determine the outload or structure of the force. In the old problem the δ_{ij} are all fixed at the outset of the problem, and they satisfy all constraints associated with them. The new problem asks the analyst to determine the best choice of δ_{ij}, i.e., the force which when allocated to the targets optimally, will maximize expected damage.

Thus, the allocation problem asks only for the set of $\left(X_{ijk}\right)$ which maximize expected value damaged given $\left(\delta_{ij}\right)$. The force structuring problem asks the analyst to determine $\left(\delta_{ij}\right)$ and $\left(X_{ijk}\right)$ which maximize the same objective function.

It may be that there are not enough warheads available to fill the force determined. The analyst then has two choices; either lower the number of weapons to be used, or lower the number of warheads in the payload types.

3.3 Discussion of Constraints.

First notice that there is a set of equations present for each launch point. These ensure that the number of warheads from weapons of a given type, allocated to its accessible targets, does not exceed the number of warheads in weapons of that type present at that launch point $\left(\delta_{ij} \cdot \alpha_j\right)$.

The δ_{ij} are restricted to integers since they represent weapons of a specific type. The restriction that

$$\sum_{j=1}^{M} \delta_{ij} = W_i \text{ for all } i$$

ensures that the capacity of launch point i is not exceeded.

Finally, the fact that if $A_{ijk} = 0$, then X_{ijk} must also equal 0, is obvious when one considers that a weapon cannot attack a target lying outside its maximum range.

There is one real-world constraint which is not considered in this model. This is concerned with the set of targets which are within range of a certain payload type from a fixed launch point; that is all K, such that $A_{ijk} = 1$ for fixed i,j. The model assumes that a weapon carrying the j^{th} payload type from the i^{th} launch point can attack any subset of its accessible targets (subject of course to the constraint that it cannot attack more targets than warheads it carries). This simplifying assumption is in general not true, because the number of targets that can be attacked depends upon such things as time of flight to the first and subsequent targets and whether fuel restrictions are exceeded. The problem concerned with the sequencing of targets to be attacked so as to remain within fuel and time of flight constraints is known as footprinting.

Once a footprint, or a set of targets to be attacked by one weapon has been determined, the Branch and Bound Algorithm,[2] developed by Little, et al., can be used to determine the least distance route through all targets in the set. The results of this algorithm when compared with maximum time of flight can be used to determine the achievability of a given footprint.

3.4 Objective Function.

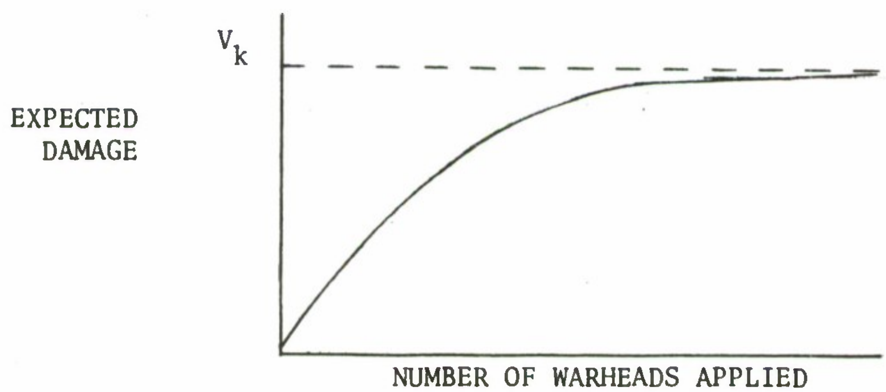
Note that the objective function G is a sum consisting of one expected damage function, f_k, associated with each target. Each f_k is a function of three variables:

1. The initial relative value of the k^{th} target, V_k.
2. The single shot kill probability of the k^{th} target, PK_k.
3. The number of warheads applied to the k^{th} target, N_k.

There are various forms of these expected damage functions; however, the following functional form is used in SEQOPT:

$$f_k = \left[1 - \left(1-PK_k\right)^{N_K}\right] \cdot V_k$$

These functions are all monatomically increasing, bounded above by V_k, and follow the law of diminishing returns.



[2]Little, J. D. C., et al., "An Algorithm for the Traveling Salesman Problem," Operations Research 11, pp. 972-989 (1963).

3.5 Possible Methods of Solution to Allocation Problems.

The body of mathematical programming techniques was first explored for a means of solving the force structuring problem. Linear programming was immediately eliminated due to the non-linearity of the objective function.

Next, dynamic programming was considered, and a solution to the force structure problem was found. Embedded within the recursive relationship of this solution is the dynamic programming solution of the standard allocation problem.

Thus, in theory, dynamic programming can be used to solve the force structuring problem; however, the memory requirements are so large that the method is at present unfeasible. (The interested reader should consult the appendix of this report for the dynamic programming formulation which solves these problems.)

Next the Branch and Bound Algorithm, developed by Little, et al.,[2] to solve the "Traveling Salesman Problem" was considered. This method divides the possible set of δ_{ij} into a tree-like structure. If a bound could be placed on these branches, large numbers of possible solutions could be eliminated. For the force structuring problem, however, no method of placing a bound on these branches could be determined and therefore the method could not be used.

Finally, the heuristic algorithm used in SEQOPT was determined by the author and tested by use of sample cases.

4. SEQOPT MODEL AND METHODS

4.1 Method Used by SEQOPT.

SEQOPT uses what may be described as a SEQuential OPTimization algorithm to solve the force structuring problem. It should be noted at the outset that the technique used in this model has no connection with

[2] Ibid.

the "Sequential Unconstrained Minimization Technique," developed by Fiacco and McCormick.[3] In fact, the method used in this model does not in general produce a true optimal solution. This fact will be illustrated later.

The sequential optimization technique derives its name from the manner in which the force structure is chosen. Simply put, the method chooses the payload type from that launch point which yields the maximum target value damaged. Thus, the method makes the best choice at each of a sequence of decision points; hence, the name sequential optimization.

Before the method is explained, several definitions (in addition to those in Section 2) are necessary.

$V_p = \left(V_{1p}, V_{2p}, \ldots, V_{TP}\right)$: a vector whose k^{th} component represents the value remaining at the k^{th} target after p weapons have been allocated to the target complex.

$D_p = \left(d_{1p}, d_{2p}, \ldots, d_{TP}\right)$: a vector whose k^{th} component represents the marginal expected damage which is achievable by placing one warhead more upon the k^{th} target, after p weapons have already been allocated.

From the foregoing definitions, it can be seen that:

$V_o = \left(V_1, V_2, \ldots, V_T\right)$; and

$D_o = \left(V_1 \cdot Pk_1, V_2 \cdot Pk_2, \ldots, V_T \cdot Pk_T\right)$.

We now define a matrix whose rows and columns correspond to payload type and launch point, respectively, and whose elements are the marginal expected value damaged possible from that launch point with a payload of that type. Let us call this matrix DAMVAL (I,J), and now we will explain how its elements are calculated.

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[3] Fiacco, A. V., and McCormick, Garth, "The Sequential Unconstrained Minimization Technique for Non-Linear Programming, A Primal-Dual Method," Management Science 10, pp. 360-366 (1964).

Suppose we wish to structure a force consisting of N weapons, and we have just chosen the payload type for the 5^{th} weapon. Define

B_K = the number of warheads which have been allocated to the k^{th} target by the first five weapons.

We must now calculate our V_5 and D_5 vectors as follows:

$$V_{k5} = V_k - \left[1-\left(PK_k\right)^{B_K}\right] V_k,$$

or simplifying

$$V_{k5} = \left(1.-PK_k\right)^{B_K} \quad V_k; \text{ for } k=1,\dots,T \text{ and}$$

$$d_{k5} = \left\{\left[1-\left(1-PK_k\right)^{B_K+1}\right] - \left[1-\left(1-PK_k\right)^{B_K}\right]\right\} \cdot V_k$$

or simplifying

$$d_{k5} = \left[\left(1-PK_k\right)^{B_K} - \left(1-PK_k\right)^{B_K+1}\right] \cdot V_k; \text{ for } k=1,\dots,T$$

$$d_{k5} = PK_k \left(1.-PK_k\right)^{B_K} \quad V_k$$

The procedure outlined below is followed in the calculation of each element of the DAMVAL matrix.

(1) Multiply the D_p vector by $\hat{A}_{ij}$, where i and j respectively correspond to the launch point and payload type we are presently considering. This has the effect of yielding no return for those targets which are not accessible.

(2) Suppose the payload we are considering contains m warheads, then we do the following m-times

a. Scan the D_p vector, selecting the target whose marginal return is the greatest;

b. Since the B_k value for the chosen target has increased by 1, the corresponding member in the D vector is updated.

(3) When all warheads for this payload type and launch point have been allocated, the total value damaged is placed in the appropriate position of the DAMVAL array.

(4) The original values of the D_p vector are replaced, and the process is repeated at (1) if the DAMVAL matrix has not been filled.

(5) When the DAMVAL matrix has been completely filled, the launch point and payload type which can contribute the maximum expected damage is chosen as a weapon to be used.

(6) The V_{p+1} and D_{p+1} vectors are recalculated and updated, and the process begins again until the total number of weapons have been given payloads and warheads have been allocated to the target complex.

An example here will help to clarify the procedure.

4.2 Example Problem-Description of Method.

Consider that in this problem we have the following data:

8 targets (T)

2 launch points (LP)

2 missile types (M)

type 1 has 2 warheads per missile (α_1)

type 2 has 3 warheads per missile (α_2)

Accessibility Matrix

	Targets							
	1	2	3	4	5	6	7	8
LP1								
Missile type 1	1	0	1	1	0	0	0	1
Missile type 2	0	0	1	1	0	0	0	0
LP2								
Missile type 1	0	1	0	1	1	1	1	0
Missile type 2	0	0	0	1	1	0	1	0

Values for targets 1 through 8 are as follows:

400., 360., 300., 280., 240., 200., 100., 50. (V-array)

The probability of kill is assumed to be .50 for each target (PK-array). We are also assuming that any payload type can be used at either launch point.

Matrix of Marginal Values*

	Targets							
	1	2	3	4	5	6	7	8
1	200.	180.	150.	140.	120.	100.	50.	25.
2	100.	90.	75.	70.	60.	50.	25.	12.5
3	50.	45.	37.5	35.	30.	25.	12.5	6.25
4	25.	22.5	18.75	17.5	15.	12.5	6.25	3.12

Define the element (i,j) of the above matrix as:

$$(i,j) = \left\{ \begin{array}{l} \text{the marginal value obtained by placing} \\ \text{i warheads on target j, over that} \\ \text{obtained by placing i-1 warheads on} \\ \text{the same target} \end{array} \right.$$

Hence, $(i,j) = [(1-pk)^{i-1} - (1-pk)^{i}]$. value of the j^{th} target.

In this example we are asked to choose four missiles, so we go through the following procedure four times:

(1) For each weapon type at each launch point, allocate the warheads to that set of accessible targets which yield the greatest marginal return. The marginal return is the sum of the individual returns for this type. (Remember to multiply the marginal return array by the appropriate accessibility array).

(2) Choose that missile, specified by launchpoint and type, which obtained the maximum marginal return. This missile is the choice for this loop.

(3) Observe the allocation of the chosen missile. Suppose that the k^{th} target was allocated two warheads, then cross out the two top entries in the k^{th} column not yet removed. The next value in this column is then the marginal return for placing one more warhead on the k^{th} target.

(4) Return to (1) if all missiles have not yet been chosen.

*The model does not store this matrix of marginal values but computes a new entry for a target whenever the number of warheads so far applied is changed.

Choice of Weapons Example

ITERATION 1

		Allocations	Marginal Value Obtained
Base 1	Type 1	1(1,3)	350.
	Type 2	2(3), 1(4)	365.*
Base 2	Type 1	1(2,4)	320.
	Type 2	2(4), 1(5)	330.

Hence, choice number 1 is Base 1 type 2.

ITERATION 2

Base 1	Type 1	2(1)	300.*
	Type 2	1(3), 2(4)	142.50
Base 2	Type 1	1(2,5)	300.*
	Type 2	1(4), 2(5)	250.

It does not matter here which choice is made because the target sets attacked by the choices have no intersection. (The next choice would certainly choose that missile not selected now.) If any of the same targets were attacked by both choices, the first type would be chosen, the previous choices and alternate would be written on tape. Then, when this example was completed using the first type as this choice, the program would then compute following choices as if the second choice here had been chosen. In this manner all possible choices are considered.

Consider choice number 2 to be Base 1 type 1.

ITERATION 3

Base	Type 1	1(1,4)	120.
	Type 2	1(3), 2(4)	142.50
Base 2	Type 1	1(2,5)	300.*
	Type 2	1(4), 2(5)	250.

Hence, choice number 3 is Base 2 type 1.

ITERATION 4		Allocations	Marginal Value Obtained
Base 1	Type 1	1(1,4)	120.
	Type 2	1(3), 2(4)	142.50
Base 2	Type 1	1(2), 1(6)	190.*
	Type 2	1(4,5,7)	180.

Hence, choice number 4 is Base 2 type 1.

The notation 1(1,3) means 1 warhead was placed on targets 1 and 3.

*Indicates weapon type and launch point selected on the iteration, and greatest marginal damage achieved.

Summarization of Outloads and Allocation

		Allocation	Marginal Expected Damage
Base 1	1 Missile type 2	2(3), 1(4)	365.00
	1 Missile type 1	2(1)	300.00
Base 2	2 Missiles type 1	1(2,5)	300.00
		1(2,6)	190.00

Target	Original Value	Expected Damage	Warheads Applied	% Value Expected Damage
1	400.	(300.)	2	.750
2	360.	(270.)	2	.750
3	300.	(225.)	2	.750
4	280.	(140.)	1	.500
5	240.	(120.)	1	.500
6	200.	(100.)	1	.500
7	100.	(0.00)	0	.000
8	50.	(0.00)	0	.000

Total value of target base - 1930.00
Total expected damage - 1155.00
% Expected damage to target base - 59.8

Note here that the choices using sequential optimization have led to a feasible solution.

4.3 Two Phases of SEQOPT.

There are two distinct phases in the operation of this model. In Phase I the constraint,

$$\sum_j \delta_{ij} = W_i ,$$

is ignored, while in Phase II this capacity constraint is observed.

The ignoring of this constraint in Phase I has the following effects:

(1) Shows the advantage, if any, of allowing the number of weapons at any launch point to exceed its real capacity;

(2) Shows the relative worth of the launch points by comparing the number of weapons actually placed there by the algorithm and its capacity;

(3) Even if the solutions in Phase I are unfeasible,* they give an upper bound to damages which could be generated in Phase II.

(4) If any solution determined in Phase I is feasible, Phase II is not executed since the results would duplicate those in Phase I.

If no feasible solution is found in Phase I, the model proceeds to Phase II where all solutions generated are feasible. This is accomplished by eliminating a launch point from further consideration once it is filled to capacity.

4.4 Multiple Results.

For a given problem, Phase I, II, or both can return multiple results. This occurs because, at each decision point where a weapon payload and launch point are chosen, the choice depends upon the maximum value in the DAMVAL matrix. When this maximum value is determined, the launch point and payload type used are chosen. There is, however, the chance that more than one element of this matrix will contain the maximum value. The following flow diagram will make clear how these tie cases are treated:

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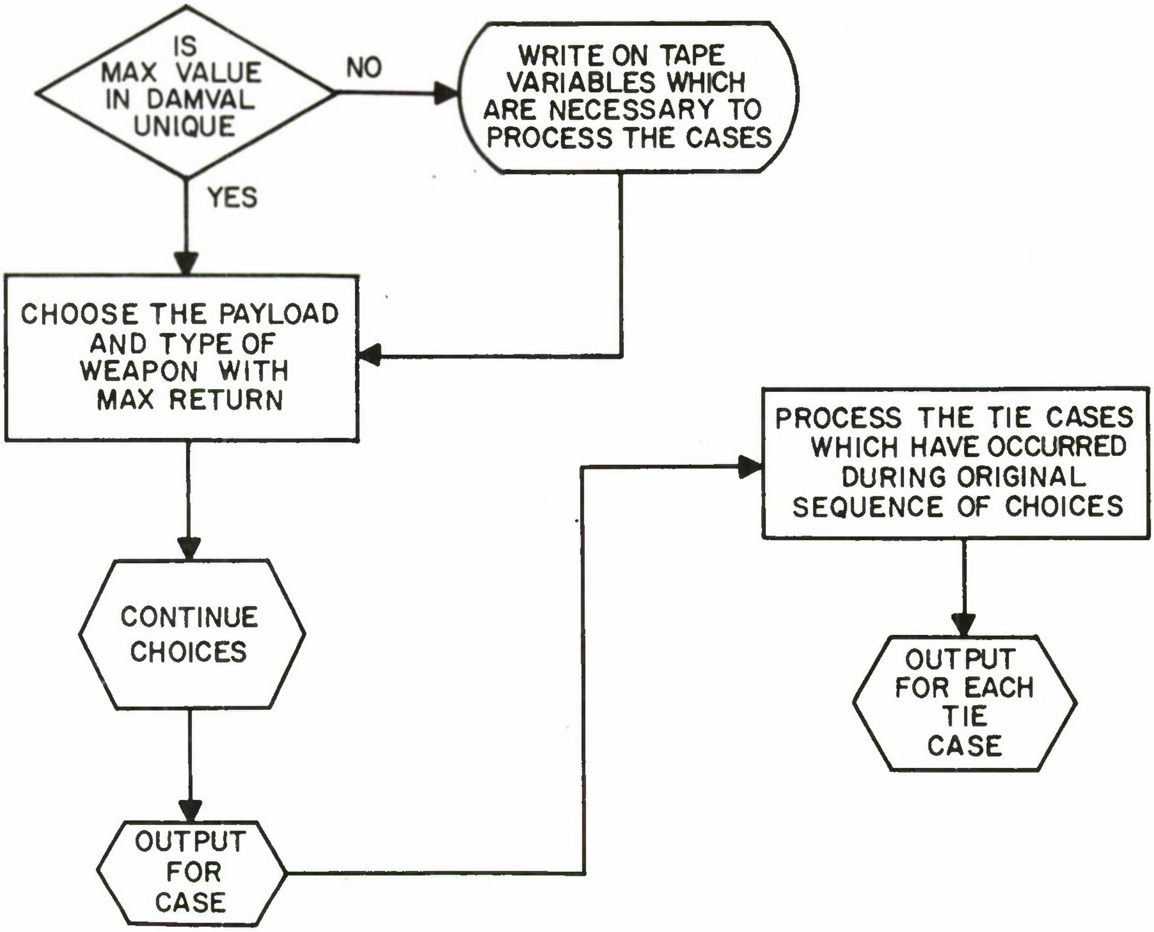
*By unfeasible, it is meant that some launch point capacity has been exceeded; all other constraints are handled by the model.



The processing of tie cases is done for the following reasons:

(1) In Phase I, no tie case can lead to an outload capable of inflicting damage greater than other outloads. A tie could, however, lead to a feasible solution, while it is possible that choosing only one alternative would not.

(2) In Phase II, tie cases could lead to higher return even though all results would be feasible.

It should be noted here that not all tie cases need to be examined. No examination of the tie is necessary when the target sets, attacked by the tying weapons, are disjoint. This is due to the fact that regardless of which weapon is chosen at this decision point, the expected damage possible for the remaining weapon will not change at the next decision point, hence, will still be maximum, and will be chosen in the next iteration.

This rationale was pointed out earlier in the example used to describe the method.

4.5 Suboptimality of Results.

Since at present there is no feasible method for finding the optimal solution to the problem while observing the constraints, we can only compare our results to an unfeasible solution, which is an upper bound, but not necessarily the least upper bound of the solution. This upper bound is computed in the following way:

(1) Remove all range constraints so every target is within range of every payload type from every launch point;

(2) Using the same D vector technique that is used in the algorithm, place one warhead at a time on that target which yields the maximum marginal return; update the marginal return from this target and continue until the maximum number of warheads have been allocated.

This method produces the greatest possible return for a specific number of warheads, although in reality it may not be feasible. By use of this method, it is possible to see how near our feasible solution comes to the unfeasible one which provides the upper bound.

At the outset, it was felt that the results of Phase I, if feasible, were an optimum solution; that is, the objective function was maximized. This, however, was shown to be false by the following counter example.[4]

Given two launch points, each with capacity of one weapon, and one payload type which has three warheads per weapon, allocate these to a set of three targets, with values V = (60, 80, 60), and accessibility matrix:

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[4]Weidman, Dr. Donald, U.S. Naval Weapons Laboratory, Dahlgren, Virginia. Conference, Naval Weapons Laboratory, Subject: SEQOPT Model, November, 1970.

	Targets		
Base	1	2	3
1	1	1	0
2	0	1	1

Also suppose that the single shot kill probability for each target is a constant .5. Then, it can be shown that SEQOPT determines the following allocation:

Target	Number Warheads Applied	Expected Value Damaged	
1	2 (1)*	45	(30)
2	3 (3)	70	(70)
3	1 (2)	30	(45)
	Total Exp. Value Damaged	145	(145)

It can also be shown that, in this case, the optimal allocation is as follows:

Target	Number Warheads Applied	Expected Value Damaged
1	2	45
2	2	60
3	2	45
	Total Exp. Value Damaged	150

The second allocation (see footnote) is as follows:

	Targets		
	1	2	3
Launch Pt 1	2	1	0
Launch Pt 2	0	1	2

The entries represent the allocation of warheads.

*The numbers in parentheses are the second allocation determined by the model. The return is exactly the same due to the symmetry of the problem.

This counter example does in fact show that the unconstrained results are not necessarily optimal. The example is, however, not the specific form which this model was designed to solve. The example is really of the standard allocation form, where no choice of force structure is possible. It is doubtful that problems where a straight allocation is needed can be handled adequately by this model. Dynamic programming models and those based on heuristic algorithms have been written for the purpose of allocating fixed forces and should be utilized for this type of problem.[1]

4.6 Other Application of Sequential Optimization Technique.

The force structuring problem addressed in this report is only one application of this sequential optimization technique. There are analogous problems in other fields such as economics where there is a limited set of resources, and they are to be distributed to maximize return. The resources present can be material or monetary; however, there are some general qualities which should be present in a problem to be solved using this technique:

(1) There must be resources of different types which could be placed at the points of distribution, so that a mixture at each point can be determined;

(2) The capacities at the distribution points must finite;

(3) Accessibility of demand to distribution points be known;

(4) Associated with each demand point there must be a return function, which follows the law of diminishing returns. The variables present in this return function are amounts of the resource types desired at this demand point;*

(5) The amount of each resource type available must be known.

---

[1] Ibid.

*It is felt that the results of this method are nearly optimal due to the form of the return functions in that they follow the law of diminishing returns.

5. PROGRAM SECTION

5.1 Input Variables.

There are six distinct data card types necessary to completely describe the variables needed for a run of this model. They are:

	COLUMNS	FORMAT	VARIABLE
Card Type 1	1-4	I4	NBASE - Number of launch points
	5-8	I4	NTYPE - Number of missile types being considered
	9-12	I4	NTARG - Number of targets in target complex
	13-42	15I2	NRESTR(I), I=1, NTYPE - Number of missiles allowed at the I^{th} launch point
Card Type 2	1-12	3I4	NDESC(I), I=1, NTYPE - Number of warheads in the I^{th} missile type
Card Type 3	1-80	80I1	{[NALLOW (I,J), J=1, NTYPE], I=1, NBASE} - 1 or 0 depending upon whether the J^{th} weapon type is allowed at the I^{th} launch point
Card Type 4	1-80	80I1	[NOYES (I,J,K), K=1, NTARG] - 1 or 0 depending upon whether the K^{th} target is within range of the J^{th} payload type from the I^{th} launch point.
Card Type 5	1-80	10F8.0	VALUE(I), I=1, NTARG - Original target values
Card Type 6	1-80	10F8.7	BPK(I), I=1, NTARG - The single shot kill probability associated with each target

5.2 Description of Important Variables: Not Covered as Input.

Variable or Array Name	Description
DELDAM (I)	Contains the marginal damage obtainable by placing one warhead more on the I^{th} target than have already been allocated there;
NCOUNT (I)	contains the number of weapons which have been selected for the I^{th} launch point;
KEEP (I,J,K)	the number of warheads of the J^{th} weapon type from the I^{th} launch point which have been allocated to the K^{th} target;
DAMAGE (I,J)	marginal damage obtainable through the use of one weapon of the J^{th} type from the I^{th} launch point;
EDSUM	accumulator for the total expected damage achievable through the use of a given outload;
NSUM (I)	the total number of warheads allocated to the I^{th} target;
ED(I)	the total expected damage to the I^{th} target for the allocation,

$$EDSUM = \sum_{I=1}^{NTARG} ED(I)$$

5.3 Running Time and Storage Limitations.

During the checkout stage of SEQOPT, cases were run whose size were such that the results could be hand verified. These cases ranged up to 25 targets and 3 launch points, and 3 missiles per launch point. All running times were less than 1 minute.

When SEQOPT had been completely checked out, a test case was devised which would compare its results with those of existing techniques used to solve the force structuring problem. This example has 200 targets, 4 launch points, and 3 missile types allowed at each launch point. The running time for this example was 10.75 minutes.

Note that all running times are for the BRLESC II, a second generation machine, and include compilation time.

It is felt that, when converted for third generation computer use, the running time can be cut in half, and the size of problems which can be handled will be greatly increased.

At present the model can handle problems with up to:

15 launch points
3 weapon types
200 targets

The storage requirement for this size problem is only 35K, which leaves a great deal of memory for expansion.

5.4 SEQOPT Program Listing.

```
APR,13,71   BRLESC2 FORTRAN.                                    CB 15..45
*      SA11WC 392 107 4483      DUMBELL

$      MAXT(30)MINS
$      MAXO(100000)LINES
       DIMENSION NUMBOM(15),NDESC(3),NALLOW(15,3),NOYES(15,3,200)
       DIMENSION VALUE(200),DELDAM(200),KATACK(45),IER(45),JER(45)
       DIMENSION NSUM(200),KEEP(15,3,200),NKEEP(15,3,200),NCOUNT(15)
       DIMENSION ED(200),DAMAGE(15,3),IIIT(200),TDAM(200),NOTED(15,3)
       DIMENSION FMT(6),FVAR(3),GMT(5),ARRAY(15,3),XRRAY(200)
       DIMENSION MOTED(15,3),NRESTR(15),BPK(200)
       DIMENSION DESC(50),LOPNOS(50)
       DATA (FMT(I),I=1,6)/10H(1H0,7HTAR,10HGET ,I3,10,4HX,5(,1H ,10H(I3,
      11H,,1X,6H),6X))/
       DATA (GMT(I),I=1,5) /8H(1H ,20X,3H,5(,1H ,8H(I3,1H,,,8H1X),6X))/
       DATA (FVAR(I),I=1,3) /1H1,1H2,1H3/
 1000  FORMAT (3I4,15I2)
 1001  FORMAT (3I4)
 1002  FORMAT (80I1)
 1003  FORMAT (10F8.0)
 1004  FORMAT (F3.2)
 1005  FORMAT (18I3)
 1006  FORMAT (F10.4)
 1007  FORMAT (I6)
 1008  FORMAT (20I4)
 1009  FORMAT (10(F5.4,3X))
 1117  FORMAT (1H0,12X,I3,14X,F6.0,10X,I2,10X,F9.2,10X,F5.3)
  102  FORMAT (1H0,33HTOTAL EXPECTED DAMAGE TO COMPLEX=,F9.2)
       LOTBOM=0
       LOWER=1
       NTRACK=1
       READ 1000, NBASE,NTYPE,NTARG,(NRESTR(I),I=1,NBASE)
       READ 1001, (NDESC(I),I=1,NTYPE)
       READ 1002, ((NALLOW(I,J),J=1,NTYPE),I=1,NBASE)
       DO 6 J=1,NTYPE
       DO 6 I=1,NBASE
       READ 1002, (NOYES(I,J,K),K=1,NTARG)
    6  CONTINUE
       DO 1 I=1,NBASE
    1  LOTBOM=LOTBOM+NRESTR(I)
       DO 5 I=1,NBASE
    5  NUMBOM(I)=LOTBOM
       READ 1003, (VALUE(I),I=1,NTARG)
       READ 1009, (BPK(I),I=1,NTARG)
       PRINT 82
   82  FORMAT (1H1,//////////,25X,45HBELOW ARE GIVEN THE OPTIMAL RETURNS
      1POSSIBLE,,/,
      225X,40HDISREGARDING ALL RANGE CONSTRAINTS, ,FOR,/,
      325X,45HALL INTEGRAL AVERAGE NUMBERS OF RV S POSSIBLE,/,
      425X,42HFOR THIS PARTICULAR SET OF TARGETS. ,THESE,/,
      525X,38HVALUES ARE UPPER BOUNDS FOR NUMBERS OF ,/,
      625X,42HRV S USED, BUT DOES NOT IMPLY THAT EXPECTED,/,
      725X,49HDAMAGE IS MONOTONICALLY INCREASING WITH RV S USED)
       PRINT 83
   83  FORMAT (1H0,/////,12X,14HAVERAGE NUMBER,8X,10HTOTAL RV S,9X,
      115HOPTIMUM ED WITH,/,13X,12HRV S/MISSILE,12X,4HUSED,10X,
      220HNO RANGE CONSTRAINTS)
       TOTKEP=0.
       DO 61 I=1,NTYPE
```

```
      DESC(I)=NDESC(I)
   61 CONTINUE
      CALL MAX1(DESC,NTYPE,MEMBER,BIG)
      CALL MIN1(DESC,NTYPE,JOK,TINY)
      MAS=BIG
      MENOS=TINY
      NROKET=LOTBOM*MAS
      DO 62 I=1,NTARG
      PK=BPK(I)
      DELDAM(I)=PK*VALUE(I)
      NKEEP(1,1,I)=0
   62 CONTINUE
      KNEBO=MAS-MENOS+1
      DO 63 I=1,KNEBO
      LOPNOS(I)=(MENOS+I-1)*LOTBOM
   63 CONTINUE
      LOWLY=1
      DO 100 I=1,KNEBO
      NBIGST=LOPNOS(I)
      DO 75 J=LOWLY,NBIGST
      CALL MAX1(DELDAM,NTARG,MEM,BIG)
      NKEEP(1,1,MEM)=NKEEP(1,1,MEM)+1
      TOTKEP=TOTKEP+BIG
      NEXP=NKEEP(1,1,MEM)+1
      MEXP=NEXP-1
      PK=BPK(MEM)
      DELDAM(MEM)=(((1.-PK)**MEXP)-((1.-PK)**NEXP))*VALUE(MEM)
   75 CONTINUE
      LOWLY=NBIGST+1
      NAV=LOPNOS(I)/LOTBOM
      PRINT 76, NAV,LOPNOS(I),TOTKEP
   76 FORMAT (1H0,17X,I5,13X,I5,13X,F10.2)
  100 CONTINUE
    4 DO 2 I=1,NTARG
      PK=BPK(I)
    2 DELDAM(I)=VALUE(I)*PK
      NWRITE=1
      NREAD=2
      LIBBY=0
      NREDO=0
      DO 3 I=1,NBASE
      NCOUNT(I)=0
      DO 3 J=1,NTYPE
      DAMAGE (I,J)=0.
      NOTED(I,J)=0
      DO 3 K=1,NTARG
      KEEP(I,J,K)=0
    3 NKEEP(I,J,K)=0
   10 DO 9000 KKKK=LOWER,LOTBOM
      DO 8000 KK=1,NBASE
      IF (NCOUNT(KK).LT.NUMBOM(KK)) GO TO 20
      DO 15 JJ=1,NTYPE
      DAMAGE(KK,JJ)=0.
      DO 15 I=1,NTARG
      NKEEP(KK,JJ,I)=0
   15 CONTINUE
      GO TO 8000
   20 DO 7000 K=1,NTYPE
      IF (NALLOW(KK,K).EQ.1) GO TO 25
```

```
      DAMAGE(KK,K)=0.
      DO 21 I=1,NTARG
   21 NKEEP(KK,K,I)=0
      GO TO 7000
   25 IF (DAMAGE(KK,K).EQ.0.) GO TO 30
      GO TO 7000
   30 DO 31 I=1,NTARG
      TEEHEE=NOYES(KK,K,I)
   31 TDAM(I)=DELDAM(I)*TEEHEE
      NER=NDESC(K)
      DO 6000 JJ=1,NER
      CALL MAX1(TDAM,NTARG,ITH,AMAX)
      DAMAGE(KK,K)=DAMAGE(KK,K)+TDAM(ITH)
      NKEEP(KK,K,ITH)=NKEEP(KK,K,ITH)+1
      NEWEXP=0
      DO 50 I=1,NBASE
      DO 50 J=1,NTYPE
   50 NEWEXP=NEWEXP+KEEP(I,J,ITH)
      NEWEXP=NEWEXP+NKEEP(KK,K,ITH)+1
      MEWEXP=NEWEXP-1
      PK=BPK(ITH)
      TDAM(ITH)=(((1.-PK)**MEWEXP)-((1.-PK)**NEWEXP))*VALUE(ITH)
 6000 CONTINUE
 7000 CONTINUE
 8000 CONTINUE
C---- ----AT THIS POINT HAVE COMPUTED FOR ALL TYPES FROM ALL BASES NOT FILLED
C---- ----             1. DAMAGE(BASE TYPE)
C---- ----             2. NKEEP(BASE,TYPE,TARGETS)
C---- ----          WILL SELECT BOMBER WITH LARGEST DAMAGE
      CALL MAXUM(DAMAGE,NBASE,NTYPE,AMIX,IER,JER,KONTER)
      IF (KONTER.EQ.1) GO TO 8444
C**** ****MATERIAL WITHIN STARS CONSIDERS TIES, WRITES DATA ON TAPES
      KZT=1
      DO 8110 KJJA=2,KONTER
      II1=IER(1)
      JJ1=JER(1)
      II2=IER(KJJA)
      JJ2=JER(KJJA)
      DO 8100 I=1,NTARG
      IF (NKEEP(II1,JJ1,I).GT.0.AND.NKEEP(II2,JJ2,I).GT.0) GO TO 8101
 8100 CONTINUE
 8101 KZT=KZT+1
      IER(KZT)=II2
      JER(KZT)=JJ2
 8110 CONTINUE
      IF(KZT.EQ.0) GO TO 8444
C---- ----HENCE KZT WILL BE THE FINAL NUMBER OF CASES PUT ON TAPE
C         ONLY THOSE CASES WHICH ATTACKED AT LEAST ONE OF THE SAME TARGETS
C---- ----AS THE CHOSEN BOMBER MUST BE REDONE
      LIBBY=LIBBY+KZT-1
      DO 8425 KJJA=2,KZT
      DO 8400 I=1,NBASE
      IF (IER(KJJA).EQ.I) GO TO 8340
      DO 8335 KOOKY=1,NTYPE
 8335 WRITE (NWRITE,1005) (KEEP(I,KOOKY,K),K=1,NTARG)
      GO TO 8400
 8340 DO 8350 J=1,NTYPE
      IF (JER(KJJA).EQ.J) GO TO 8345
      WRITE (NWRITE,1005)  (KEEP(I,J,K),K=1,NTARG)
```

```
      GO TO 8350
 8345 DO 8346 K=1,NTARG
 8346 IIIT(K)=NKEEP(I,J,K)+KEEP(I,J,K)
      WRITE (NWRITE,1005)  (IIIT(K),K=1,NTARG)
 8350 CONTINUE
 8400 CONTINUE
      KK=IER(KJJA)
      JJ=JER(KJJA)
      DO 8415 I=1,NTARG
      IF (NKEEP(KK,JJ,I).EQ.0) GO TO 8413
      KSUM=NKEEP(KK,JJ,I)
      DO 8410 J=1,NBASE
      DO 8410 L=1,NTYPE
 8410 KSUM=KSUM+KEEP(J,L,I)
      KSUM=KSUM+1
      MSUM=KSUM-1
      PK=BPK(I)
      TTT=(((1.-PK)**MSUM)-(1.-PK)**KSUM)*VALUE(I)
      WRITE (NWRITE,1006) TTT
      GO TO 8415
 8413 WRITE (NWRITE,1006) DELDAM(I)
 8415 CONTINUE
      DO 8423 I=1,NBASE
      IF (I.EQ.IER(KJJA)) GO TO 8421
      WRITE (NWRITE,1007) NCOUNT(I)
      GO TO 8423
 8421 NTD=NCOUNT(I)+1
      WRITE (NWRITE,1007) NTD
 8423 CONTINUE
      KTO=KKKK+1
      IF (KTO.LE.LOTBOM) GO TO 8422
      KTO=999999
 8422 WRITE (NWRITE,1007) KTO
      NACK=IER(KJJA)
      MACK=JER(KJJA)
      DO 8424 I=1,NBASE
      DO 8424 J=1,NTYPE
      IF (I.EQ.NACK.AND.J.EQ.MACK) GO TO 8420
      MOTED(I,J)=NOTED(I,J)
      GO TO 8424
 8420 MOTED(I,J)=NOTED(I,J)+1
 8424 CONTINUE
      WRITE (NWRITE,1008) ((MOTED(I,J),J=1,NTYPE),I=1,NBASE)
 8425 CONTINUE
C**** ****  CASE DATA FOR ALL TIES NOW WRITTEN ON TAPE UNIT 8.
 8444 ITE=IER(1)
      JTE=JER(1)
      NOTED(ITE,JTE)=NOTED(ITE,JTE)+1
      NCOUNT(ITE)=NCOUNT(ITE)+1
      DO 8500 I=1,NTARG
      IF (NKEEP(ITE,JTE,I).EQ.0) GO TO 8500
      KEEP(ITE,JTE,I)=KEEP(ITE,JTE,I)+NKEEP(ITE,JTE,I)
      NZRA=0
      DO 8499 J=1,NBASE
      DO 8499 K=1,NTYPE
 8499 NZRA=NZRA+KEEP(J,K,I)
      NZRA=NZRA+1
      MZRA=NZRA-1
      PK=BPK(I)
```

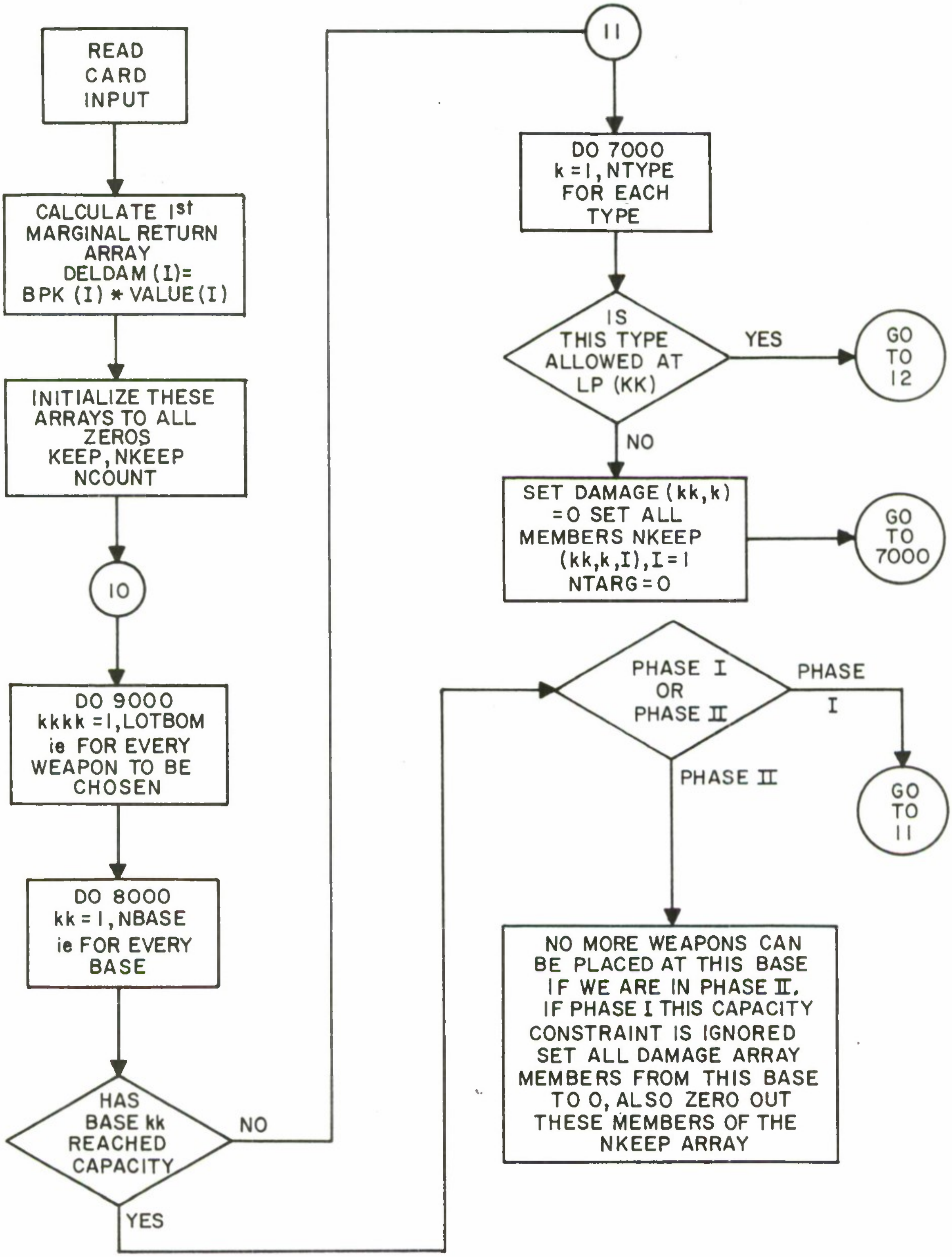
```
      DELDAM(I)=(((1.-PK)**MZRA)-(1.-PK)**NZRA)*VALUE(I)
 8500 CONTINUE
C---- ----IF ANY TYPES NOT CHOSEN ATTACKED ANY TARGETS ALSO ATTACKED BY
C          THE CHOSEN TYPE, THEIR CONTRIBUTIONS ARE SET TO ZERO,AND WILL
C                                BE RECALCULATED
      KKOUNT=1
      DO 8600 I=1,NTARG
      IF (NKEEP(ITE,JTE,I).EQ.0) GO TO 8600
      KATACK(KKOUNT)=I
      KKOUNT=KKOUNT+1
 8600 CONTINUE
      KKOUNT=KKOUNT-1
      DO 8800 I=1,NBASE
      DO 8780 J=1,NTYPE
      DO 8750 K=1,KKOUNT
      KTH=KATACK(K)
      IF (NKEEP(I,J,KTH).EQ.0) GO TO 8750
      DAMAGE(I,J)=0.
      DO 8749 KB=1,NTARG
 8749 NKEEP(I,J,KB)=0
      GO TO 8780
 8750 CONTINUE
 8780 CONTINUE
 8800 CONTINUE
 9000 CONTINUE
C---- ---- WHEN EXIT FROM THIS LOOP HAVE ASSIGNED ALL BOMBERS
C                   ALL PERMANENT ASSIGNMENTS ARE IN KEEP ARRAY
 9010 EDSUM=0
      DO 9100 I=1,NTARG
      NSUM(I)=0
      DO 9050 J=1,NBASE
      DO 9050 K=1,NTYPE
      NSUM(I)=NSUM(I)+KEEP(J,K,I)
 9050 CONTINUE
      NOOO=NSUM(I)
      PK=BPK(I)
      ED(I)=(1.-(1.-PK)**NOOO)*VALUE(I)
      EDSUM=EDSUM+ED(I)
 9100 CONTINUE
C---- ----             NOW BEGIN OUTPUT FOR THIS CASE
      IF (NTRACK.EQ.2) GO TO 9116
      DO 9110 I=1,NBASE
      IF (NCOUNT(I).GT.NRESTR(I)) GO TO 9115
 9110 CONTINUE
      PRINT 10110
10110 FORMAT (1H1,20X,45H*****THIS SOLUTION YIELDS OPTIMAL RETURN*****)
      CHECK=1.
      GO TO 9116
 9115 PRINT 10119
10119 FORMAT (1H1,20X,40H*****THIS IS AN UNFEASIBLE SOLUTION*****)
 9116 PRINT 10111
10111 FORMAT (1H0,65HBELOW ARE THE NUMBERS OF WEAPONS OF EACH TYPE CHOSE
     1N AT EACH BASE)
      DO 9200 M=1,NBASE
      PRINT 10121
10121 FORMAT (1H0)
      PRINT 1014, M, NUMBOM(M)
 1014 FORMAT(1H ,5HBASE ,I2,27H NUMBER OF WEAPONS ALLOWED=,I2)
      DO 9190 I=1,NTYPE
```

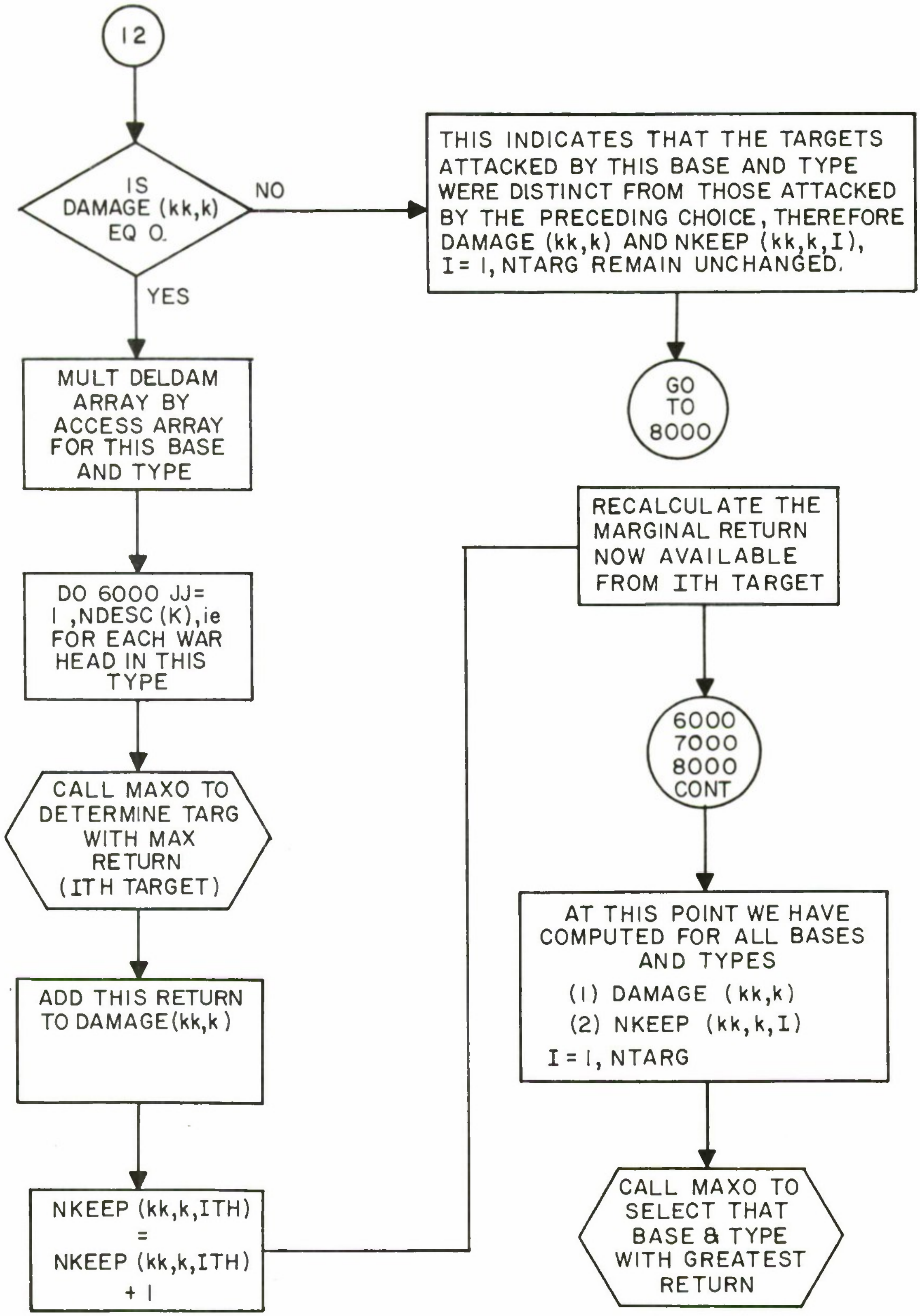
```
      PRINT 1013, I,NOTED(M,I)
 1013 FORMAT(1H0,5HTYPE ,I1,2H =,I2)
 9190 CONTINUE
 9200 CONTINUE
      PRINT 8062
 8062 FORMAT(1H0,91HBELOW IS A LISTING OF TARGETS,NUMBER OF WARHEADS APP
     1LIED, AND VALUE BEFORE AND AFTER ATTACK)
      PRINT 1015
 1015 FORMAT (1H0,15X,6HTARGET,8X,8HORIGINAL,8X,9HNUMBER OF,8X,
     18HEXPECTED,8X,13HPERCENTAGE OF)
      PRINT 1016
 1016 FORMAT (30X,5HVALUE,6X,16HWARHEADS APPLIED,6X,6HDAMAGE,9X,
     113HVALUE DAMAGED)
      PRINT 10121
      DO 11111 I=1,NTARG
      RRINTD=ED(I)/VALUE(I)
      PRINT 1117,I,VALUE(I),NSUM(I),ED(I),RRINTD
11111 CONTINUE
      PRINT 102, EDSUM
      PRINT 1017
 1017 FORMAT (1H1,73HBELOW IS THE ALLOCATION OF WEAPONS TO TARGETS, SEPA
     1RATED BY BASE AND TYPE)
      FMT(4)=FVAR(NTYPE)
      GMT(3)=FVAR(NTYPE)
      DO 1333 I=1,NTARG
      IF (NBASE.GT.5) GO TO 1323
      NYO=NBASE
      GO TO 1324
 1323 NYO=5
 1324 WRITE (6,FMT) I,((KEEP(J,K,I),K=1,NTYPE),J=1,NYO)
      IF (NBASE.LE.5) GO TO 1333
      IF (NBASE.LE.10) GO TO 1326
      NYO=10
      GO TO 1327
 1326 NYO=NBASE
 1327 WRITE (6,GMT)     ((KEEP(J,K,I),K=1,NTYPE),J=6,NYO)
      IF (NBASE.LE.10) GO TO 1333
      NYO=NBASE
      WRITE (6,GMT)    ((KEEP(J,K,I),K=1,NTYPE),J=11,NYO)
 1333 CONTINUE
      IF (NREDO.EQ.0) GO TO 1500
C**** *********THIS IS READ IN SECTION FROM TAPE NREAD
 1400 DO 1410 I=1,NBASE
      DO 1410 J=1,NTYPE
 1410 READ (NREAD,1005) (KEEP(I,J,K),K=1,NTARG)
      DO 1420 I=1,NBASE
      DO 1420 J=1,NTYPE
      DAMAGE(I,J)=0.
      DO 1420 K=1,NTARG
 1420 NKEEP(I,J,K)=0
      DO 1425 I=1,NTARG
 1425 READ (NREAD,1006) DELDAM(I)
      DO 1430 J=1,NBASE
 1430 READ (NREAD,1007) NCOUNT(J)
      READ (NREAD,1007) LOWER
      READ (NREAD,1008) ((NOTED(I,J),J=1,NTYPE),I=1,NBASE)
C**** *********THIS ENDS THE READ IN SECTION
      NREDO=NREDO-1
      IF (LOWER.EQ.999999) GO TO 9010
```

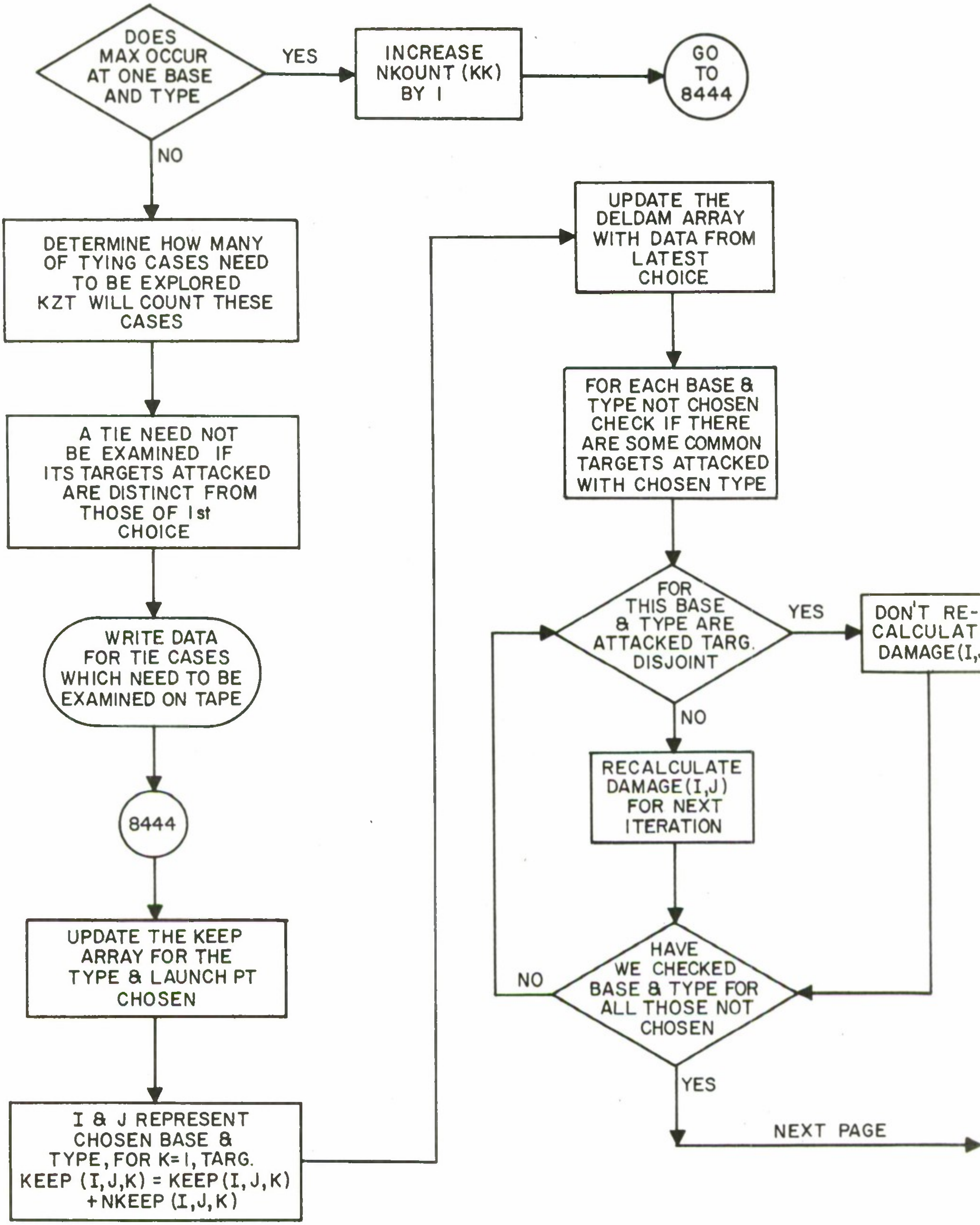
```
      GO TO 10
 1500 IF (LIBBY.EQ.0) GO TO 1600
      REWIND NREAD
      REWIND NWRITE
      NREDO=LIBBY
      LIBBY=0
      NREAD=NWRITE
      IF (NREAD.EQ.1) GO TO 1502
      NWRITE=1
      GO TO 1505
 1502 NWRITE=2
 1505 GO TO 1400
 1600 CONTINUE
      IF (NTRACK.EQ.2) GO TO 1700
      IF (CHECK.EQ.0.) GO TO 1610
      PRINT 1601
 1601 FORMAT (1H1,15X,89H*****AN OPTIMAL FEASIBLE SOLUTION HAS BEEN FOUN
     1D, FURTHER CALCULATION IS UNNECESSARY*****)
      GO TO 1700
 1610 LOWER =1
      DO 1612 I=1,NBASE
 1612 NUMBOM(I)=NRESTR(I)
      NTRACK=2
      REWIND NREAD
      REWIND NWRITE
      PRINT 1699
 1699 FORMAT (1H1,//////////,20X,80HNO OPTIMAL FEASIBLE SOLUTION HAS BEE
     1N FOUND-ALL FOLLOWING SOLUTIONS ARE FEASIBLE)
      GO TO 4
 1700 CONTINUE
      STOP
      END
      SUBROUTINE MAXUM(ARRAY,NROW,NCOL,AMIX,IER,JER,KONTER)
      DIMENSION ARRAY(15,3),IER(5),JER(5)
      KONTER=1
      AMIX=0.
      DO 100 I=1,NROW
      DO 90 J=1,NCOL
      IF (ARRAY(I,J).LT.AMIX) GO TO 90
      IF (ARRAY(I,J).GT.AMIX) GO TO 80
      IER(KONTER)=I
      JER(KONTER)=J
      KONTER=KONTER+1
      GO TO 90
   80 AMIX=ARRAY(I,J)
      IER(1)=I
      JER(1)=J
      KONTER=2
      DO 87 K=2,5
      IER(K)=0
   87 JER(K)=0
   90 CONTINUE
  100 CONTINUE
      KONTER=KONTER-1
      RETURN
      END
      SUBROUTINE MAX1(XRRAY,NSIZE,MEMBER,THEMAX)
      DIMENSION XRRAY(200)
      MEMBER =1
```

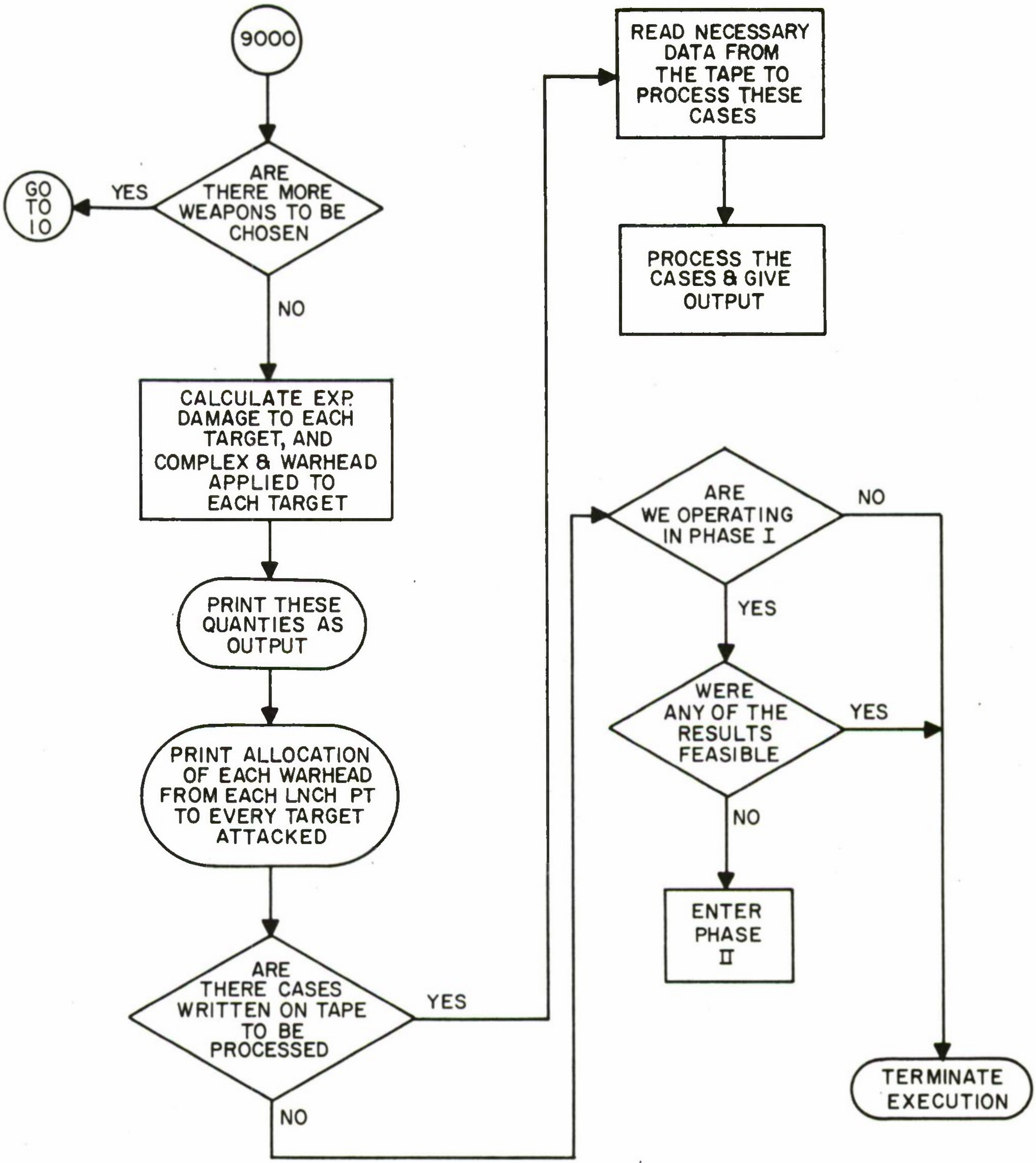
```
      THEMAX=XRRAY(1)
      DO 10 I=2,NSIZE
      IF (XRRAY(I).LE.THEMAX) GO TO 10
      THEMAX=XRRAY(I)
      MEMBER=I
   10 CONTINUE
      RETURN
      END
      SUBROUTINE MIN1(ARRAY,NSIZE,MEMBER,THEMIN)
      DIMENSION ARRAY(200)
      MEMBER=1
      THEMIN=ARRAY(1)
      DO 10 I=2,NSIZE
      IF (ARRAY(I).GE.THEMIN) GO TO 10
      MEMBER=2
      THEMIN=ARRAY(I)
   10 CONTINUE
      RETURN
      END
      LIST
*     LIST(STOP)
```

5.5 SEQOPT Flow Charts.




12
IS DAMAGE (kk,k) EQ O.
NO
YES
THIS INDICATES THAT THE TARGETS ATTACKED BY THIS BASE AND TYPE WERE DISTINCT FROM THOSE ATTACKED BY THE PRECEDING CHOICE, THEREFORE DAMAGE (kk,k) AND NKEEP (kk,k,I), I = 1, NTARG REMAIN UNCHANGED.
GO TO 8000
MULT DELDAM ARRAY BY ACCESS ARRAY FOR THIS BASE AND TYPE
DO 6000 JJ= 1 ,NDESC (K), ie FOR EACH WAR HEAD IN THIS TYPE
CALL MAXO TO DETERMINE TARG WITH MAX RETURN (ITH TARGET)
ADD THIS RETURN TO DAMAGE(kk,k)
NKEEP (kk,k,ITH) = NKEEP (kk,k,ITH) + 1
RECALCULATE THE MARGINAL RETURN NOW AVAILABLE FROM ITH TARGET
6000 7000 8000 CONT
AT THIS POINT WE HAVE COMPUTED FOR ALL BASES AND TYPES
(1) DAMAGE (kk,k)
(2) NKEEP (kk,k,I)
I = 1, NTARG
CALL MAXO TO SELECT THAT BASE & TYPE WITH GREATEST RETURN


DOES MAX OCCUR AT ONE BASE AND TYPE
YES
INCREASE NKOUNT (KK) BY 1
GO TO 8444
NO
DETERMINE HOW MANY OF TYING CASES NEED TO BE EXPLORED KZT WILL COUNT THESE CASES
A TIE NEED NOT BE EXAMINED IF ITS TARGETS ATTACKED ARE DISTINCT FROM THOSE OF 1st CHOICE
WRITE DATA FOR TIE CASES WHICH NEED TO BE EXAMINED ON TAPE
8444
UPDATE THE KEEP ARRAY FOR THE TYPE & LAUNCH PT CHOSEN
I & J REPRESENT CHOSEN BASE & TYPE, FOR K=1, TARG. KEEP (I,J,K) = KEEP(I,J,K) +NKEEP (I,J,K)
UPDATE THE DELDAM ARRAY WITH DATA FROM LATEST CHOICE
FOR EACH BASE & TYPE NOT CHOSEN CHECK IF THERE ARE SOME COMMON TARGETS ATTACKED WITH CHOSEN TYPE
FOR THIS BASE & TYPE ARE ATTACKED TARG. DISJOINT
YES
DON'T RE-CALCULATE DAMAGE(I,J)
NO
RECALCULATE DAMAGE(I,J) FOR NEXT ITERATION
HAVE WE CHECKED BASE & TYPE FOR ALL THOSE NOT CHOSEN
NO
YES
NEXT PAGE


9000
ARE THERE MORE WEAPONS TO BE CHOSEN
YES
GO TO 10
NO
CALCULATE EXP. DAMAGE TO EACH TARGET, AND COMPLEX & WARHEAD APPLIED TO EACH TARGET
PRINT THESE QUANTIES AS OUTPUT
PRINT ALLOCATION OF EACH WARHEAD FROM EACH LNCH PT TO EVERY TARGET ATTACKED
ARE THERE CASES WRITTEN ON TAPE TO BE PROCESSED
YES
NO
READ NECESSARY DATA FROM THE TAPE TO PROCESS THESE CASES
PROCESS THE CASES & GIVE OUTPUT
ARE WE OPERATING IN PHASE I
NO
YES
WERE ANY OF THE RESULTS FEASIBLE
YES
NO
ENTER PHASE II
TERMINATE EXECUTION

IBM FORTRAN Coding Form

X28-7327-6 U/M050
Printed in U.S.A.

PROGRAM SEQOPT

PROGRAMMER SET-UP FOR SAMPLE PROBLEM

DATE | PUNCHING INSTRUCTIONS | GRAPHIC | PUNCH | PAGE OF | CARD ELECTRO NUMBER*

STATEMENT NUMBER | CONT. | FORTRAN STATEMENT | IDENTIFICATION SEQUENCE

```
   3  2  10 4 4 4        (NBASE,NTYPE,NTARG,NRESTR(I),I=1,NBASE)
   3  2                  (NDESC(I),I=1,NTYPE)
111111                   (NALLOW(I,J),J=1,NTYPE),I=1,NBASE)
0011011000 }
1111011001 }
0100111001 }  ACCESSABILITY
1100111101 }  MATRIX
1100100110 }
1101101110 }               (TARGET VALUES)
     300.     290.     260.     260.     210.     200.     190.     150.     100.     100.
 .33     .42     .40     .38     .35     .30     .45     .50     .30     .37
                  (SINGLE SHOT KILL PROBABILITIES)
```

5.6 Sample Problem Description.

Suppose we have a target complex which consists of ten targets, with the following relative values and single shot kill probabilities

	Targets									
	1	2	3	4	5	6	7	8	9	10
VALUE	300	290	260	260	210	200	190	150	100	100
PK	.33	.42	.40	.38	.35	.30	.45	.50	.30	.37

We also have three launch points each of which has a capacity of four weapons. We will consider two possible weapon types (payloads).

Type I = 3 warheads/missile

Type II = 2 warheads/missile

The accessibility matrices are as defined below:

		Targets									
No. 1 Launch Point		1	2	3	4	5	6	7	8	9	10
	Type 1	0	0	1	1	0	1	1	0	0	0
	Type 2	1	1	1	1	0	1	1	0	0	1
No. 2 Launch Point											
	Type 1	0	1	0	0	1	1	1	0	0	1
	Type 2	1	1	0	0	1	1	1	1	0	1
No. 3 Launch Point											
	Type 1	1	1	0	0	1	0	0	1	1	0
	Type 2	1	1	0	1	1	0	1	1	1	0

5.7 SEQOPT Output For This Example.

Each iteration of this model gives output which can be divided into three distinct portions; these contain:

(1) The capacity of each launch point and also the number of each weapon type chosen for each launch point are given. (Note that in Phase I the capacity of each launch point is equal to the total number of weapons to be chosen; if Phase II is used, the capacities are reset to their true values.)

(2) The listing of targets along with their original values, number of warheads applied to each target, expected damage, and percentage of target value damaged. Finally, the total expected damage for the target complex is given.

(3) The allocation of warheads to targets, separated by base and type. Turning to the first output of the sample case, the first line is interpreted as follows:

Base 1: types 1 and 2 had no warheads delivered to target 1.

Base 2: type 1 delivered 4 warheads to target 1, type 2 delivered 0.

Base 3: types 1 and 2 had no warheads delivered to target 1.

Notice that Phase I runs are terminated by the statement: "NO OPTIMAL FEASIBLE SOLUTION HAS BEEN FOUND - ALL FOLLOWING SOLUTIONS ARE FEASIBLE," and all output following is from Phase II.

BELOW ARE GIVEN THE OPTIMAL RETURNS POSSIBLE, DISREGARDING ALL RANGE CONSTRAINTS, ,FOR ALL INTEGRAL AVERAGE NUMBERS OF RV S POSSIBLE FOR THIS PARTICULAR SET OF TARGETS. ,THESE VALUES ARE UPPER BOUNDS FOR NUMBERS OF RV S USED, BUT DOES NOT IMPLY THAT EXPECTED DAMAGE IS MONOTONICALLY INCREASING WITH RV S USED

AVERAGE NUMBER RV S/MISSILE	TOTAL RV S USED	OPTIMUM ED WITH NO RANGE CONSTRAINTS
2	24	1441.19
3	36	1704.71

*****THIS IS AN UNFEASIBLE SOLUTION*****

BELOW ARE THE NUMBERS OF WEAPONS OF EACH TYPE CHOSEN AT EACH BASE

BASE 1 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 0

TYPE 2 = 1

BASE 2 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 7

TYPE 2 = 1

BASE 3 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 3

TYPE 2 = 0

BELOW IS A LISTING OF TARGETS,NUMBER OF WARHEADS APPLIED, AND VALUE BEFORE AND AFTER ATTACK

TARGET	ORIGINAL VALUE	NUMBER OF WARHEADS APPLIED	EXPECTED DAMAGE	PERCENTAGE OF VALUE DAMAGED
1	300.	4	239.55	.798
2	290.	4	257.18	.887
3	260.	4	226.30	.870
4	260.	4	221.58	.852
5	210.	4	172.51	.821
6	200.	4	151.98	.760
7	190.	3	158.39	.834
8	150.	2	112.50	.750
9	100.	2	51.00	.510
10	100.	3	75.00	.750

TOTAL EXPECTED DAMAGE TO COMPLEX= 1665.99

BELOW IS THE ALLOCATION OF WEAPONS TO TARGETS, SEPARATED BY BASE AND TYPE

TARGET	1	0,	0,	4,	0,	0,	0,
TARGET	2	0,	0,	4,	0,	0,	0,
TARGET	3	0,	0,	4,	0,	0,	0,
TARGET	4	0,	0,	4,	0,	0,	0,
TARGET	5	0,	0,	0,	0,	4,	0,
TARGET	6	0,	0,	1,	0,	3,	0,
TARGET	7	0,	0,	3,	0,	0,	0,
TARGET	8	0,	2,	0,	0,	0,	0,
TARGET	9	0,	0,	0,	2,	0,	0,
TARGET	10	0,	0,	1,	0,	2,	0,

*****THIS IS AN UNFEASIBLE SOLUTION*****

BELOW ARE THE NUMBERS OF WEAPONS OF EACH TYPE CHOSEN AT EACH BASE

BASE 1 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 0

TYPE 2 = 0

BASE 2 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 7

TYPE 2 = 2

BASE 3 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 3

TYPE 2 = 0

BELOW IS A LISTING OF TARGETS,NUMBER OF WARHEADS APPLIED, AND VALUE BEFORE AND AFTER ATTACK

TARGET	ORIGINAL VALUE	NUMBER OF WARHEADS APPLIED	EXPECTED DAMAGE	PERCENTAGE OF VALUE DAMAGED
1	300.	4	239.55	.798
2	290.	4	257.18	.887
3	260.	4	226.30	.870
4	260.	4	221.58	.852
5	210.	4	172.51	.821
6	200.	4	151.98	.760
7	190.	3	158.39	.834
8	150.	2	112.50	.750
9	100.	2	51.00	.510
10	100.	3	75.00	.750

TOTAL EXPECTED DAMAGE TO COMPLEX= 1665.99

BELOW IS THE ALLOCATION OF WEAPONS TO TARGETS, SEPARATED BY BASE AND TYPE.

TARGET	1	0,	0,	4,	0,	0,	0,
TARGET	2	0,	0,	4,	0,	0,	0,
TARGET	3	0,	0,	4,	0,	0,	0,
TARGET	4	0,	0,	4,	0,	0,	0,
TARGET	5	0,	0,	0,	0,	4,	0,
TARGET	6	0,	0,	1,	0,	3,	0,
TARGET	7	0,	0,	3,	0,	0,	0,
TARGET	8	0,	0,	0,	2,	0,	0,
TARGET	9	0,	0,	0,	2,	0,	0,
TARGET	10	0,	0,	1,	0,	2,	0,

*****THIS IS AN UNFEASIBLE SOLUTION*****

BELOW ARE THE NUMBERS OF WEAPONS OF EACH TYPE CHOSEN AT EACH BASE

BASE 1 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 0

TYPE 2 = 0

BASE 2 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 7

TYPE 2 = 1

BASE 3 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 3

TYPE 2 = 1

BELOW IS A LISTING OF TARGETS,NUMBER OF WARHEADS APPLIED, AND VALUE BEFORE AND AFTER ATTACK

TARGET	ORIGINAL VALUE	NUMBER OF WARHEADS APPLIED	EXPECTED DAMAGE	PERCENTAGE OF VALUE DAMAGED
1	300.	4	239.55	.798
2	290.	4	257.18	.887
3	260.	4	226.30	.870
4	260.	4	221.58	.852
5	210.	4	172.51	.821
6	200.	4	151.98	.760
7	190.	3	158.39	.834
8	150.	2	112.50	.750
9	100.	2	51.00	.510
10	100.	3	75.00	.750

TOTAL EXPECTED DAMAGE TO COMPLEX= 1665.99

BELOW IS THE ALLOCATION OF WEAPONS TO TARGETS, SEPARATED BY BASE AND TYPE.

TARGET 1	0,	0,	4,	0,	0,	0,
TARGET 2	0,	0,	4,	0,	0,	0,
TARGET 3	0,	0,	4,	0,	0,	0,
TARGET 4	0,	0,	4,	0,	0,	0,
TARGET 5	0,	0,	0,	0,	4,	0,
TARGET 6	0,	0,	1,	0,	3,	0,
TARGET 7	0,	0,	3,	0,	0,	0,
TARGET 8	0,	0,	0,	0,	0,	2,
TARGET 9	0,	0,	0,	2,	0,	0,
TARGET 10	0,	0,	1,	0,	2,	0,

*****THIS IS AN UNFEASIBLE SOLUTION*****

BELOW ARE THE NUMBERS OF WEAPONS OF EACH TYPE CHOSEN AT EACH BASE

BASE 1 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 0

TYPE 2 = 1

BASE 2 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 7

TYPE 2 = 0

BASE 3 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 3

TYPE 2 = 1

BELOW IS A LISTING OF TARGETS,NUMBER OF WARHEADS APPLIED, AND VALUE BEFORE AND AFTER ATTACK

TARGET	ORIGINAL VALUE	NUMBER OF WARHEADS APPLIED	EXPECTED DAMAGE	PERCENTAGE OF VALUE DAMAGED
1	300.	4	239.55	.798
2	290.	4	257.18	.887
3	260.	4	226.30	.870
4	260.	4	221.58	.852
5	210.	4	172.51	.821
6	200.	4	151.98	.760
7	190.	3	158.39	.834
8	150.	2	112.50	.750
9	100.	2	51.00	.510
10	100.	3	75.00	.750

TOTAL EXPECTED DAMAGE TO COMPLEX= 1665.99

BELOW IS THE ALLOCATION OF WEAPONS TO TARGETS, SEPARATED BY BASE AND TYPE.

TARGET	1	0,	0,	4,	0,	0,	0,
TARGET	2	0,	0,	4,	0,	0,	0,
TARGET	3	0,	0,	4,	0,	0,	0,
TARGET	4	0,	0,	4,	0,	0,	0,
TARGET	5	0,	0,	0,	0,	4,	0,
TARGET	6	0,	0,	1,	0,	3,	0,
TARGET	7	0,	0,	3,	0,	0,	0,
TARGET	8	0,	2,	0,	0,	0,	0,
TARGET	9	0,	0,	0,	0,	0,	2,
TARGET	10	0,	0,	1,	0,	2,	0,

*****THIS IS AN UNFEASIBLE SOLUTION*****

BELOW ARE THE NUMBERS OF WEAPONS OF EACH TYPE CHOSEN AT EACH BASE

BASE 1 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 0

TYPE 2 = 0

BASE 2 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 7

TYPE 2 = 1

BASE 3 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 3

TYPE 2 = 1

BELOW IS A LISTING OF TARGETS,NUMBER OF WARHEADS APPLIED, AND VALUE BEFORE AND AFTER ATTACK

TARGET	ORIGINAL VALUE	NUMBER OF WARHEADS APPLIED	EXPECTED DAMAGE	PERCENTAGE OF VALUE DAMAGED
1	300.	4	239.55	.798
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3	260.	4	226.30	.870
4	260.	4	221.58	.852
5	210.	4	172.51	.821
6	200.	4	151.98	.760
7	190.	3	158.39	.834
8	150.	2	112.50	.750
9	100.	2	51.00	.510
10	100.	3	75.00	.750

TOTAL EXPECTED DAMAGE TO COMPLEX= 1665.99

BELOW IS THE ALLOCATION OF WEAPONS TO TARGETS, SEPARATED BY BASE AND TYPE

TARGET	1	0,	0,	4,	0,	0,	0,
TARGET	2	0,	0,	4,	0,	0,	0,
TARGET	3	0,	0,	4,	0,	0,	0,
TARGET	4	0,	0,	4,	0,	0,	0,
TARGET	5	0,	0,	0,	0,	4,	0,
TARGET	6	0,	0,	1,	0,	3,	0,
TARGET	7	0,	0,	3,	0,	0,	0,
TARGET	8	0,	0,	0,	2,	0,	0,
TARGET	9	0,	0,	0,	0,	0,	2,
TARGET	10	0,	0,	1,	0,	2,	0,

*****THIS IS AN UNFEASIBLE SOLUTION*****

BELOW ARE THE NUMBERS OF WEAPONS OF EACH TYPE CHOSEN AT EACH BASE

BASE 1 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 0

TYPE 2 = 0

BASE 2 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 7

TYPE 2 = 0

BASE 3 NUMBER OF WEAPONS ALLOWED=12

TYPE 1 = 3

TYPE 2 = 2

BELOW IS A LISTING OF TARGETS,NUMBER OF WARHEADS APPLIED, AND VALUE BEFORE AND AFTER ATTACK

TARGET	ORIGINAL VALUE	NUMBER OF WARHEADS APPLIED	EXPECTED DAMAGE	PERCENTAGE OF VALUE DAMAGED
1	300.	4	239.55	.798
2	290.	4	257.18	.887
3	260.	4	226.30	.870
4	260.	4	221.58	.852
5	210.	4	172.51	.821
6	200.	4	151.98	.760
7	190.	3	158.39	.834
8	150.	2	112.50	.750
9	100.	2	51.00	.510
10	100.	3	75.00	.750

TOTAL EXPECTED DAMAGE TO COMPLEX= 1665.99

BELOW IS THE ALLOCATION OF WEAPONS TO TARGETS, SEPARATED BY BASE AND TYPE

TARGET 1	0,	0,	4,	0,	0,	0,
TARGET 2	0,	0,	4,	0,	0,	0,
TARGET 3	0,	0,	4,	0,	0,	0,
TARGET 4	0,	0,	4,	0,	0,	0,
TARGET 5	0,	0,	0,	0,	4,	0,
TARGET 6	0,	0,	1,	0,	3,	0,
TARGET 7	0,	0,	3,	0,	0,	0,
TARGET 8	0,	0,	0,	0,	0,	2,
TARGET 9	0,	0,	0,	0,	0,	2,
TARGET 10	0,	0,	1,	0,	2,	0,

NO OPTIMAL FEASIBLE SOLUTION HAS BEEN FOUND-ALL FOLLOWING SOLUTIONS ARE FEASI

BELOW ARE THE NUMBERS OF WEAPONS OF EACH TYPE CHOSEN AT EACH BASE

BASE 1 NUMBER OF WEAPONS ALLOWED= 4

TYPE 1 = 2

TYPE 2 = 2

BASE 2 NUMBER OF WEAPONS ALLOWED= 4

TYPE 1 = 4

TYPE 2 = 0

BASE 3 NUMBER OF WEAPONS ALLOWED= 4

TYPE 1 = 3

TYPE 2 = 1

BELOW IS A LISTING OF TARGETS,NUMBER OF WARHEADS APPLIED, AND VALUE BEFORE AND AFTER ATTACK

TARGET	ORIGINAL VALUE	NUMBER OF WARHEADS APPLIED	EXPECTED DAMAGE	PERCENTAGE OF VALUE DAMAGED
1	300.	4	239.55	.798
2	290.	4	257.18	.887
3	260.	4	226.30	.870
4	260.	5	236.18	.908
5	210.	4	172.51	.821
6	200.	4	151.98	.760
7	190.	3	158.39	.834
8	150.	2	112.50	.750
9	100.	1	30.00	.300
10	100.	2	60.31	.603

TOTAL EXPECTED DAMAGE TO COMPLEX= 1644.91

BELOW IS THE ALLOCATION OF WEAPONS TO TARGETS, SEPARATED BY BASE AND TYPE

TARGET 1	0,	1,	3,	0,	0,	0,
TARGET 2	0,	1,	2,	0,	1,	0,
TARGET 3	2,	0,	2,	0,	0,	0,
TARGET 4	3,	0,	2,	0,	0,	0,
TARGET 5	0,	1,	0,	0,	3,	0,
TARGET 6	1,	0,	1,	0,	2,	0,
TARGET 7	0,	0,	2,	0,	1,	0,
TARGET 8	0,	1,	0,	0,	0,	1,
TARGET 9	0,	0,	0,	0,	0,	1,
TARGET 10	0,	0,	0,	0,	2,	0,

BELOW ARE THE NUMBERS OF WEAPONS OF EACH TYPE CHOSEN AT EACH BASE

BASE 1 NUMBER OF WEAPONS ALLOWED= 4

TYPE 1 = 3

TYPE 2 = 1

BASE 2 NUMBER OF WEAPONS ALLOWED= 4

TYPE 1 = 4

TYPE 2 = 0

BASE 3 NUMBER OF WEAPONS ALLOWED= 4

TYPE 1 = 3

TYPE 2 = 1

BELOW IS A LISTING OF TARGETS,NUMBER OF WARHEADS APPLIED, AND VALUE BEFORE AND AFTER ATTACK

TARGET	ORIGINAL VALUE	NUMBER OF WARHEADS APPLIED	EXPECTED DAMAGE	PERCENTAGE OF VALUE DAMAGED
1	300.	4	239.55	.798
2	290.	4	257.18	.887
3	260.	5	239.78	.922
4	260.	5	236.18	.908

5	210.	3	152.33	.725
6	200.	5	166.39	.832
7	190.	4	172.61	.908
8	150.	2	112.50	.750
9	100.	0	0.00	.000
10	100.	2	60.31	.603

TOTAL EXPECTED DAMAGE TO COMPLEX= 1636.83

BELOW IS THE ALLOCATION OF WEAPONS TO TARGETS, SEPARATED BY BASE AND TYPE

TARGET 1	0,	1,	3,	0,	0,	0,
TARGET 2	0,	0,	2,	0,	1,	1,
TARGET 3	3,	0,	2,	0,	0,	0,
TARGET 4	3,	0,	2,	0,	0,	0,
TARGET 5	0,	0,	0,	0,	3,	0,
TARGET 6	2,	0,	1,	0,	2,	0,
TARGET 7	1,	0,	2,	0,	1,	0,
TARGET 8	0,	1,	0,	0,	0,	1,
TARGET 9	0,	0,	0,	0,	0,	0,
TARGET 10	0,	0,	0,	0,	2,	0,

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APPENDIX

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DYNAMIC PROGRAMMING SOLUTION TO FORCE STRUCTURING PROBLEM

To determine the dynamic programming solution to this problem, it is first necessary to formulate the solution to the standard allocation problem. First, some additional definitions are needed:

(1) Associated with each target there is a return function f_i, which is a monotonically increasing bounded function. The bound of this function is the target value V_i, hence

$$V_i = \operatorname*{Sup}_{N} [f_i (V_i, PK_i, N)].$$

In other words V_i is the limiting return obtainable from the i^{th} target regardless of the number of warheads applied.

(2) Let V be the vector whose components are the original values of the targets.

(3) Let ΔV be a vector whose components represent the remaining values of the targets after some or all have been attacked. Suppose N_i warheads have been placed on the i^{th} target, then

$$\Delta V_i = V_i - f_i (V_i, PK_i, N_i).$$

(4) Let X be a two-dimensional array whose elements X_{ij} are defined as follows:

X_{ij} = the number of warheads of the j^{th} payload type placed at the i^{th} launch point.

Suppose that some warheads are allocated. Then let the matrix ΔX represent the unused warheads.

ΔX_{ij} = the number of warheads of the j^{th} payload type at the i^{th} launch point, not yet allocated.

Note that by specifying the X matrix the Force Structure is automatically determined.

(5) Now let G represent the set of targets in the complex, so

$$G: \ (t_1, t_2, \ldots, t_T).$$

Define a function H as follows:

$$H_k\ (X,V,G) = \begin{cases} \text{The return gained by distributing the} \\ \text{resources in X to any set of K targets,} \\ \text{using an optimal policy, where the} \\ \text{targets are chosen from G, having values} \\ \text{given in the V vector.} \end{cases}$$

The recursive relationship is:

$$H_k(X,V,G) = \max_{k = 1,2,\ldots,t} \ \max_{\substack{0 \leq X_{11K} \leq X_{11} \\ 0 \leq X_{12K} \leq X_{12} \\ \vdots \quad \vdots \quad \vdots \\ 0 \leq X_{LP,M,K} \leq X_{LP,M}}} \left[f_k(V_k, PK_k, \sum_{I=1}^{LP} \sum_{J=1}^{M} X_{ijk}) + H_{k-1}\ (\Delta X,\ \Delta V,\ G - t_k) \right]$$

We know that $H_o\ (X,V,G) = 0$, since applying weapons to no targets yields no return.

With this formulation we are now ready to obtain the solution to the force structuring problem. In this formulation we have a two-dimensional state variable:

$$\text{State} = (V,\ Y)\ ,$$

where Y is defined as follows:

$$Y = (y_1,\ y_2, \ldots, y_n)\ ,$$

where y_i is the total number of weapons of the i^{th} type which are available for use. Y is, therefore, the resource vector.

Our stage variable will represent the number of launch points to which we are presently assigning weapons. Also, δ_{ij}, will have the same definition as given in the earlier description, that of defining the structure of the force.

Define our relationship now as:

$L_k(Y,V)$ = the maximum return obtainable from the set of targets for an optimal choice and allocation of forces from k launch points given a resource vector Y and target values, V^o.

The recursion formula for this relationship is:

$$L_k(Y,V) = \max_{\substack{0 \leq \delta k1 \leq y_1 \\ 0 \leq \delta k2 \leq y_2 \\ 0 \leq \delta km \leq y_m}} \left\{ [H_T \, (y_{k1} \cdot N_1, \ldots, y_{km} \cdot N_m; V, G) + L_{k-1}(\Delta Y, \Delta V)] \right\}$$

It can be noted here that the solution of the standard allocation problem is embedded in the solution of the force structuring problem. For each iteration in the structuring problem, a complete allocation problem must be solved. For a problem of even moderate size the computing time becomes prohibitive due to the large dimensionality of the state variables.

Thus, dynamic programming yields a solution, in theory, to the force structuring problem. However, the use of this method is impractical due to time and memory constraints.

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Target complex

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13. ABSTRACT

SEQOPT has been designed to solve a specific force structuring and allocation problem. Given the following:

1. A set of targets each with a relative value and an associated single shot kill probability;

2. A set of launch points for weapons and the number of weapons allowed at each launch point;

3. The payload types available for use in these weapons;

4. The set of accessible targets for a weapon carrying each payload type from every launch point.

SEQOPT will then determine the payload type for each weapon in the force, and the allocation of each weapon to its set of accessible targets, in a way which attempts to maximize the expected damage to the target complex.

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Force structure						
Allocation						
Target complex						

Kill prob